

**Education
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Schools' Mathematics Consultancy,
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A Foray into Reasoning & Proof

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Continuing Professional
Development
Standard

National Centre
for Excellence in the
Teaching of Mathematics



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Our Workshop

▶ **Part 1 Reasoning**

▶ **Part 2 Proof**

Rationale

▶ The overarching aims of the National Curriculum are :

*Develop fluency * Reason mathematically *Solve problems

▶ At KS3/4 Reason Mathematically includes :

▶ By following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language:

▶ pupils begin to reason deductively in geometry, number and algebra, including using geometrical constructions...”

▶ To be able to build a proof - we need to develop reasoning.

Aims

1. To begin to understand how to develop Reasoning across the curriculum
2. To de-mystify Proof
3. To broaden your perception of the topic to encompass a softer approach and to feel really comfortable with it.
4. I would like you to see proof as a tangible element of the syllabus, rather than an abstract and difficult topic.

TASK on Reasoning

- ▶ 2 Exercises - working in pairs or triples
- ▶ Keep in mind - what skill set are we using to solve these problems?
- ▶ What will students find difficult? What errors/misconceptions might they have?
- ▶ Report back shortly!

Part 1 : Reasoning

► Exercise 1

$$T = \sqrt{\frac{w}{d^3}}$$

$$w = 5.6 \times 10^{-5}$$

$$d = 1.4 \times 10^{-4}$$

(a) Work out the value of T .

Give your answer in standard form correct to 3 significant figures.

Part 1 : Reasoning

Exercise 1 ctd

w is increased by 10%

d is increased by 5%

Lottie says,

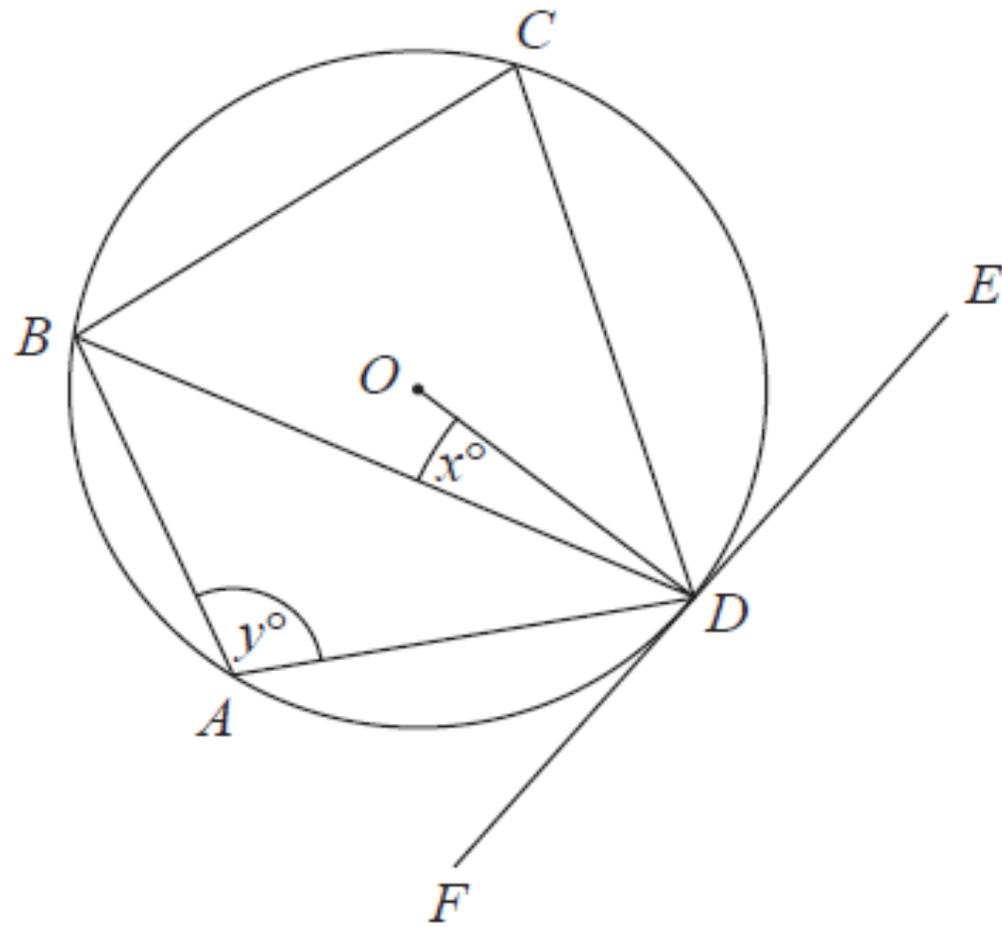
“The value of T will increase because both w and d are increased.”

(b) Lottie is wrong.

Explain why.

Part 1 : Reasoning

► Exercise 2



A , B , C and D are points on the circumference of a circle, centre O .
 FDE is a tangent to the circle.

(a) Show that $y - x = 90$

You must give a reason for each stage of your working.

Part 1 : Reasoning

Exercise 2 ctd

Dylan was asked to give some possible values for x and y .

He said,

“ y could be 200 and x could be 110, because $200 - 110 = 90$ ”

(b) Is Dylan correct?

You must give a reason for your answer.

Discussion in 2's or 3's

- ▶ Skill set we are using : how can we develop this?
- ▶ What will students find difficult? What are the errors/misconceptions?
- ▶ And what are the answers???
- ▶ Feedback shortly -write comments on post-it notes

Your Feedback on Reasoning

Plenary on Reasoning

- ▶ It's clear what we mean by *Reasoning*.
- ▶ *It's* a long-term process - But *it's* a vital pre-cursor to developing proof.
- ▶ Practical Suggestions
- ▶ 1. Developing logical reasoning
- ▶ Keep asking Why?
- ▶ Using logic -a way of aligning and organising thoughts,
- ▶ Explain to your neighbour (peer teaching),
- ▶ Critique your neighbours justification;
- ▶ Learning how to use a representation for a question (eg diagram, algebra) will assist reasoning skills,
- ▶ Encourage pupils not to be afraid to make mistakes,
- ▶ Tell a story - all based around deductive logic - the students can only ask you questions to which you can Answer YES or NO.

- ▶ 2. Scaffolding :
- ▶ At the end of each topic - do some questions on reasoning using the topic content.
(Handout on this F/H Levels).

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Part 2 : Proof

- ▶ Overview :
- ▶ Contextualise Proof into everyday life
- ▶ **Activity KS3**
 - ▶ Discussion and feedback/Reflections

Part 2 Proof

Proof is

- ▶ A fancy argument
- ▶ A justification to show a claim is 100% TRUE
- ▶ We cannot disprove anything - we can find examples where a result does not hold or fails (counterexample)
- ▶ Let's look at some real-life examples - proof can be seen every day in our lives

Contextualise Proof into everyday life

Proof - Example 1

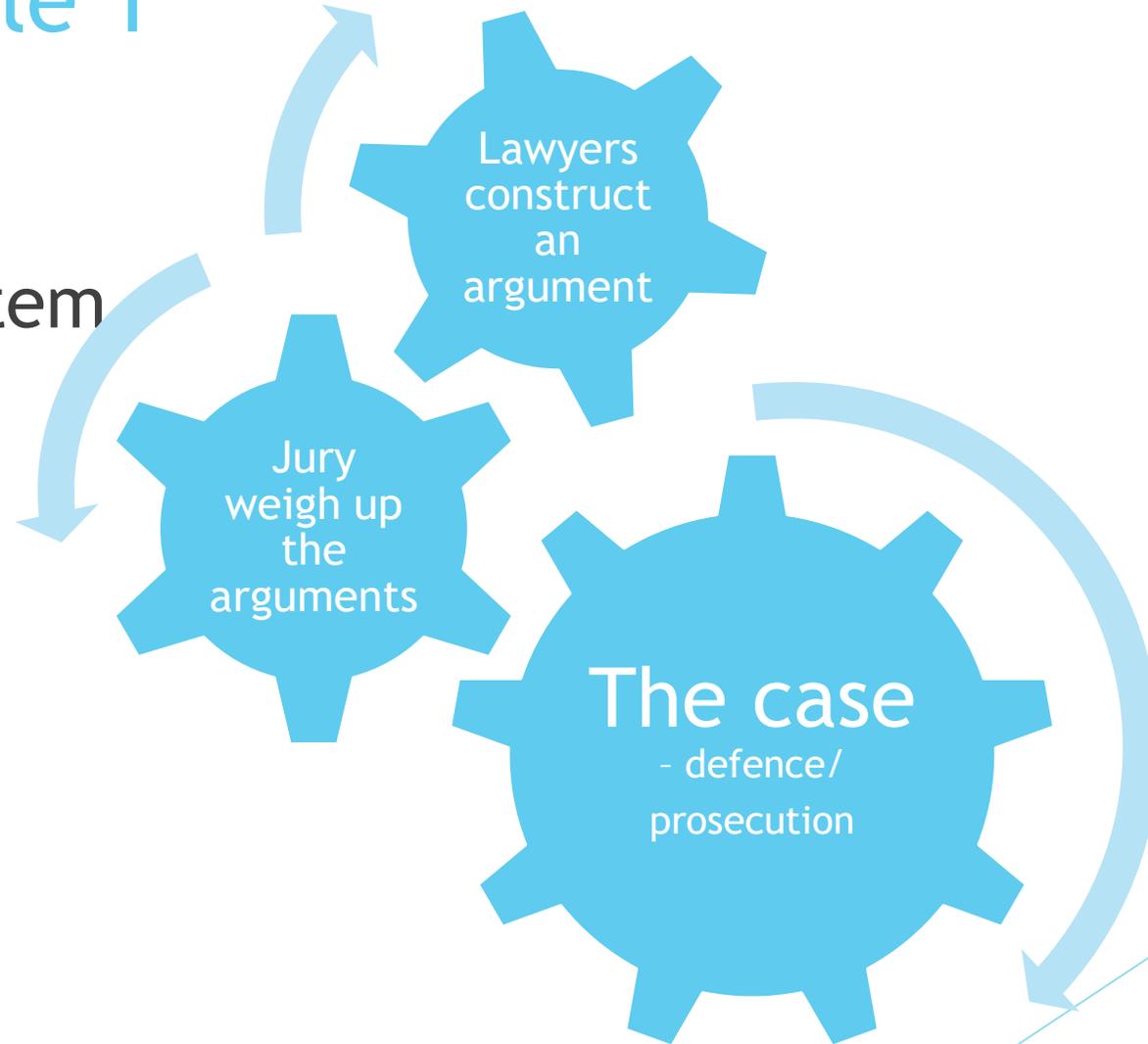
► The Justice System



Contextualise Proof into everyday life

Proof - Example 1

► The Justice System



Proof - Example 1

- ▶ Lawyers construct an argument using the evidence (logical, deductive....)
- ▶ Not 100% proof (known as “proving beyond reasonable doubt”)
- ▶ In Maths we always have to be 100% certain that our claim is true

Proof - Example 2 -Debating Society

Your School's Debating Society debate the claim :

TOPIC : Our is Climate Changing. Discuss.

Contextualise Proof into everyday life

**Argument
For**

- Extreme weather events occur more frequently
- Polar caps are melting

**Argument
Against**

- Over the long-term there are fluctuations in climate
- Random events do occur

VOTE

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Proof in Mathematics

A Proof is a

- ▶ A structured argument
- ▶ It is irrefutable
- ▶ Might use Number properties and structures and our understanding of these
- ▶ Might use algebra, Geometry, proportions.....
- ▶ Whenever we ask “why?”, “justify your results”, “explain your answer”, “decide if something is true” these are all pre-cursors to Proof

Proof in Mathematics

- ▶ Reading task -
- ▶ Please read summary of research article on proof
- ▶ Read : Analysing & Categorising proofs (p2)

Proofs - checking if a result holds true using Number

Example 3

Show that the sum of 2 consecutive integers is always odd

Firstly let's just check it out (try a few numbers)

Eg 3,4

$$\text{Sum} = 3 + 4 = 7$$



ODD

6, 7

$$\text{Sum} = 6 + 7 = 13$$



ODD

102, 103,

$$\text{Sum} = 102 + 103 = 205$$



ODD

CONCLUSION : YES I CAN BELIEVE ITS TRUE - next stage is to formalise this.

Proofs

- ▶ Activity - in Pairs or Triples
- ▶ KS3 Mathematics Matching Exercise True or False
- ▶ Categorise the problems

Proofs - For KS3 Discussion in 3's

Write each point on a post-it note

- ▶ What would pupils find difficult?
- ▶ What do you find is difficult to teach?
- ▶ What would be your pupils errors or misconceptions?
- ▶Feedback shortly

Feedback from teachers from previous workshops

- ▶ The square root concept - that it usually is a decimal
- ▶ Key words - they don't always understand the vocabulary - factor, multiple.
- ▶ They find it difficult changing fractions into decimals.
- ▶ Understanding fractions in general.
- ▶ They mix up the operations Squaring and Square Rooting,
- ▶ Understanding Multiple, factor - they find this difficult.

Scaffolding ideas KS3

Some Practical Suggestions :

Learning to translate English into Mathematics needs to be firmly embedded early into KS3.

- ▶ Language - of number needs to be very familiar. Knowledge of the Number systems, structures and properties of numbers needs to be explored and developed from KS3 to KS4.
 - Knowledge of multiple.
- ▶ Vocabulary used in proof needs to become natural to students - counterexample, show, prove....
- ▶ Developing logical reasoning - keep asking **Why? Explain to your neighbour (peer teaching), critique your neighbours justification.**

Scaffolding ideas

KS4

- **Some Practical Suggestions :**
- Looking at how to start a proof -
- Learning how to represent a question (eg diagram, algebra)
- How to use logic - aligning and organising thoughts
- Encourage pupils not to be afraid to make mistakes
- Continue to develop logical reasoning - keep asking **Why? Explain to your neighbour (peer teaching), critique your neighbours justification.**